

## Lesson 17: EGYPTIAN NUMERATION

This lesson is intended to help the children understand our place values in the decimal system of numeration and to make a comparison with an ancient system of numeration where the numerals had no place value. The children should not memorize the Egyptian system. The lesson should be omitted for slow learners.

The ancient Egyptian system was based on two principles -- repetition and addition. It was a decimal system (system of tens), but was primarily used for recording purposes and not for computation. The Egyptian had a set of basic symbols, of which only three are presented to the children:

a vertical stroke		to represent 1
a heel bone	∩	to represent 10
a scroll	9	to represent 100

Ten vertical strokes equaled one heel bone; ten heel bones equaled one scroll, etc. If the Egyptian wished to write a numeral for one item, he drew one vertical stroke (|); for two items, two vertical strokes (||); for three items, three vertical strokes (|||); and so on, up to nine strokes for nine items (|||||||).

When the ancient Egyptian wished to write or represent ten, he used one heel bone (∩); for twenty, he used two heel bones (∩∩); for thirty, three heel bones (∩∩∩), etc. He did the same kind of repetition when writing scrolls for hundreds -- four hundred would be represented by four scrolls (9999). The four scrolls would represent  $100 + 100 + 100 + 100$ . The Egyptian number, 99∩|, would represent  $200 + 10 + 1$  or 211.

You may wish to begin the lesson with a discussion of ancient Egypt. If so, pictures and films would be helpful. A list of books containing suitable pictures for display is included in the materials list for this lesson.

## MATERIALS

- pictures of Egypt (optional)

Recommended sources are:

Life Magazine, April 1968 issues

The River Nile by Bruce Brander, National Geographic Society, Washington, D.C., 1966

Dictionary of Egyptian Civilization by Georges Posener, Tudor Publishing Co., New York, 1959

Thebes of the Pharaohs by Charles F. Nims. Stein and Day, New York, 1965

Life in the Ancient World by Bart Winer, Random House 1961, pp. 34-77

The Golden Book of Lost Worlds by Janet Chenery, Golden Press, New York 1963, pp. 7-49

- Worksheets 42, 43 and 44

## PROCEDURE

Tell your class that Tor must have spent some time at the library reading books about number systems other than ours, because he has mentioned some of the numerals used by the ancient Egyptians. If you have pictures or films about ancient Egypt, show them to the children. Pictures of the Sphinx, the Nile River, and of the wall paintings the ancient Egyptians made would be appropriate. Also show the children where Egypt is on a map or globe. Say that Tor thought the Egyptians must have been very interesting and he would like the children to hear about the Egyptian numerals.

Before starting the presentation of the Egyptian numerals, ask the children whether any of them knows what our number system is called. If no child does, provide the answer -- the Hindu-Arabic system.

### Activity A

Explain to the children the meaning of the Egyptian symbols, |, n, and 9, i.e., that they represent one, ten, and one

hundred, respectively. Point out that the Egyptians had no symbol to represent zero. Write 17 on the chalk board. Call on a child to come to the chalk board and write the Egyptian numeral for 17. (17 |||||.)

Help the child write the correct answer. Continue the activity by writing several examples on the chalk board and repeating the same procedure. Use numerals such as 6, 32, 94, 368, 416, etc. Have the children practice writing these numerals at their desks while the others are writing on the chalk board. Help the children complete Worksheets 42, 43, and 44.

Worksheet 42		Unit 16		Name _____	
= 1	= 10	= 100			
Write the following in Egyptian numerals.					
32 =					
74 =					
142 =					
483 =					
774 =					
444 =					

Worksheet 43		Unit 16		Name _____	
= 1	= 10	= 100			
Write the following in our numerals.					
	=	734			
	=	286			
	=	403			
	=	475			
	=	255			

Worksheet 44  
Unit 16

Name: \_\_\_\_\_

Write the sums in Egyptian, then change to our numerals.

Example:

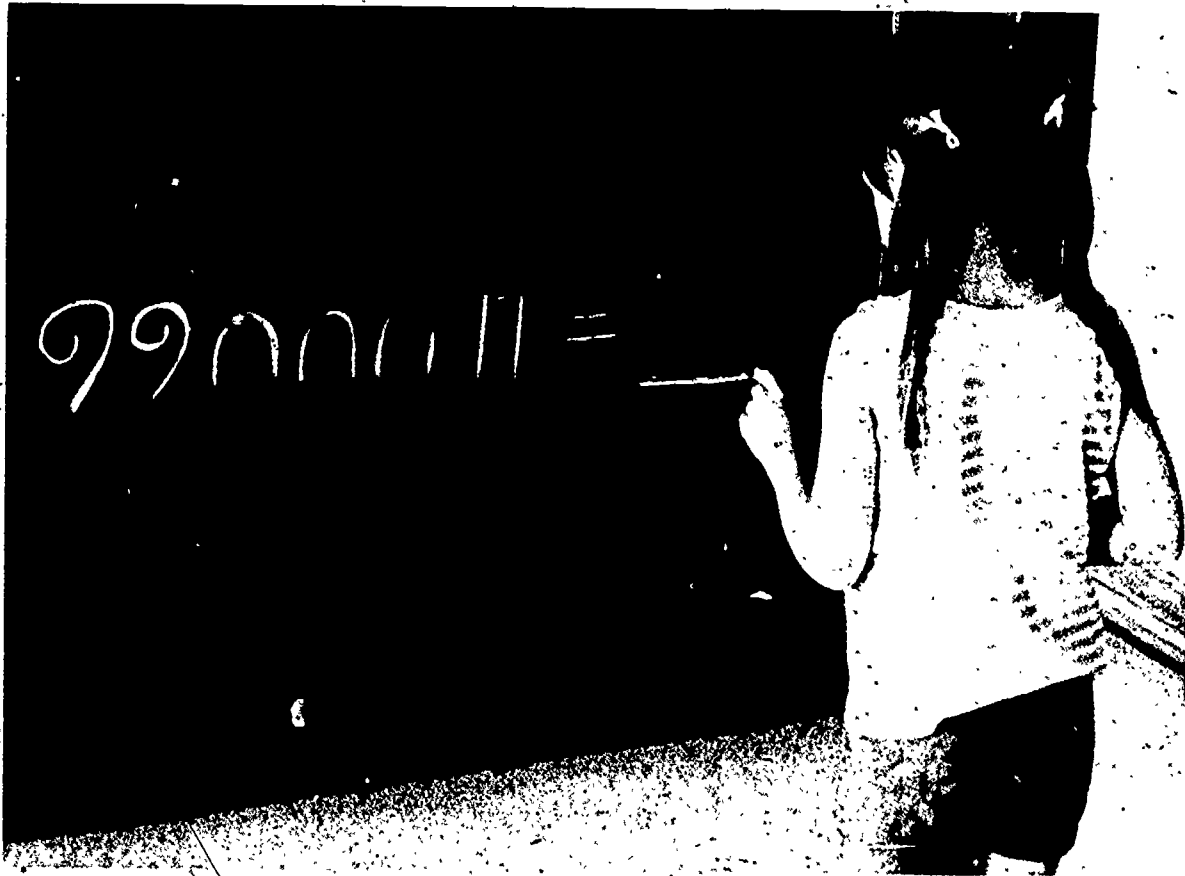
$$20|| + 20|| = 2020|||| = 40$$

$$999|| + 99| = 999999||| = 333$$

$$99| + 99| = 9999|| = 222$$

$$99999 + |||| = 999999|||| = 504$$

$$99 + 999 + ||| = 999999||| = 233$$



Activity B

Write on the chalk board:

$$462 = 9999\text{nnnnnnn}||$$

Then ask:

HOW MANY DIGITS OR SEPARATE SYMBOLS DID WE HAVE TO USE TO WRITE 462 IN OUR SYSTEM? (3.)

HOW MANY IN THE EGYPTIAN SYSTEM? (12.)

WHICH IS EASIER TO USE? (Our system.)

Show the children that the numeral 462 may be written in any order. It can be written, for example as:

$$9999\text{nnnnnnn}|| \text{ or } \text{nnnn}999999||$$

Explain that this is because the Egyptian system uses only the properties of addition and repetition. It follows no order and the numerals have no place value.

If you wish, repeat with other examples, such as:

$$962 \text{ and } 9999999999\text{nnnnnnn}||$$

## Lesson 18: ROMAN NUMERATION

A brief investigation into the Roman numeration system should enable a child to appreciate our decimal (base ten) system more. Roman numerals are still used frequently in our society. You can see them on clock faces, in books, on cornerstones, in motion picture screen credits, in English theme outlines, and so on. Little emphasis should be placed on mastering the reading of Roman numerals -- the emphasis should be on comparing decimal numerals with Roman numerals. The children will see that the Roman system is more complex.

As with the previous lesson, you may wish to show pictures that tell the children something about the life of the ancient Romans. Three books containing suitable pictures are listed in the materials for this lesson.

### MATERIALS

- pictures of ancient Rome (optional)

Recommended sources are:

Life in the Ancient World by Bart Winer, Random House, Inc., 1961.

Ancient Rome by Roger Butterfield, Odyssey Press, New York, 1964.

The Fountains of Rome by H. V. Morton, MacMillan Company, New York, 1966.

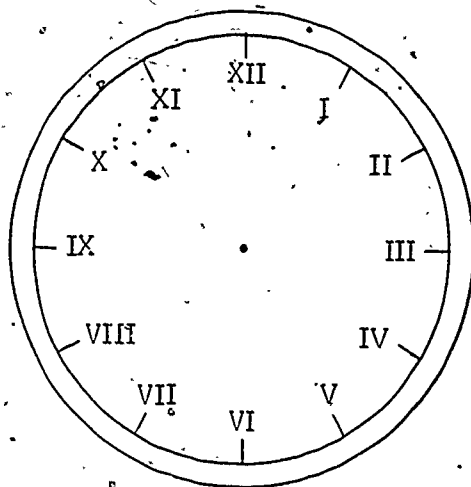
- Worksheets 45, 46 and 47

### PROCEDURE

If you wish, tell the children that Tor has now been reading about the ancient Roman numeration system and would like the class to take a look at these numerals. As an introduction, discuss briefly some of the history of ancient Rome, showing pictures of Caesar, the Colosseum, etc. Have the children join in this discussion with information from films they have seen, too.

### Activity A

Draw a large picture of a clock face on a piece of tagboard.  
Use Roman numerals to label it.



HAS ANYONE EVER SEEN A CLOCK FACE THAT LOOKS  
LIKE THIS? WHERE?

WHAT KIND OF NUMERALS ARE THESE? (Roman.)

WHERE ELSE HAVE YOU SEEN THESE NUMERALS? (In books,  
on buildings, etc.)

WHAT ARE THE DIGITS OR SYMBOLS THAT APPEAR ON THIS  
CLOCK FACE? (I, V, X.)

LET'S SEE IF WE CAN FIGURE OUT WHAT THESE SYMBOLS  
MEAN. MAYBE WE CAN COMPARE THIS CLOCK WITH OUR  
REGULAR CLOCK.

Draw a picture of a regular clock face and place it along side  
of the Roman numeral clock face.

Have the children look at Worksheet 45. Now, by one-to-one correspondence, compare the symbols as follows:

WHAT DOES THE I ON THE ROMAN CLOCK REPRESENT? (1.)

Draw a line from 1 to I. Write the I in the correct square.

WHAT DOES THE V ON THE ROMAN CLOCK REPRESENT? (5.)

Draw a line from 5 to V. Write the V in the correct square.

WHAT DOES THE X ON THE ROMAN CLOCK REPRESENT? (10.)

Draw a line from 10 to X. Write the X in the correct square.

Continue comparing all the numerals on the clock faces. Give the IV and the IX last.

CAN YOU EXPLAIN THE IV? (Four, implied as one before five.)

CAN YOU EXPLAIN THE IX? (Nine, implied as one before ten.)

Have the children complete Worksheet 45 by placing the corresponding numerals in the proper spaces. Remember to explain the notation as you proceed; e.g., XII = 10 + 1 + 1 = 12, etc.

Worksheet 45  
Unit 16 Name \_\_\_\_\_

Hindu-Arabic Roman

Complete this chart.

Hindu-Arabic	1	2	3	4	5	6
Roman	I	II	III	IV	V	VI
Hindu-Arabic	7	8	9	10	11	12
Roman	VII	VIII	IX	X	XI	XII



Continue by drawing the symbols "L" and "C" on your chalk board. Explain that these symbols are Roman numerals which represent 50 and 100, respectively. Ask the children to identify orally XXXIII, IX, CXX, IX, etc.

Then have them complete Worksheets 46 and 47.

Worksheet 46  
Unit 16 Name \_\_\_\_\_

Write the Roman numerals for:

1 = <u>I</u>	15 = <u>XV</u>
2 = <u>II</u>	20 = <u>XX</u>
4 = <u>IV</u>	50 = <u>L</u>
6 = <u>VI</u>	65 = <u>LXV</u>
8 = <u>VIII</u>	69 = <u>LXIX</u>
9 = <u>IX</u>	90 = <u>XC</u>
10 = <u>X</u>	100 = <u>C</u>
13 = <u>XIII</u>	200 = <u>CC</u>

Worksheet 47  
Unit 16 Name \_\_\_\_\_

Write our numerals for:

VII = <u>7</u>	IX = <u>9</u>
XVII = <u>17</u>	XIX = <u>19</u>
XXII = <u>22</u>	LVI = <u>56</u>
IV = <u>4</u>	CCXX = <u>220</u>
XXIV = <u>24</u>	CCC = <u>300</u>

### Activity B

Write 33 and XXXIII on the chalk board.

HOW MANY DIGITS OR SYMBOLS FOR 33 DO WE USE IN OUR SYSTEM? (2.)

HOW MANY IN THE ROMAN SYSTEM? (6.)

WHICH IS EASIER TO WRITE? (33, the Hindu-Arabic.)

DOES THE ROMAN NUMERAL HAVE PLACE VALUE? (No.)

Explain that it does have the property of order, i.e., XXXIII must be written exactly as it is. It cannot be written XXIXI. The Roman system, as the Egyptian system, does not have a symbol for zero.

Repeat with:

L and 50

LV and 55

XXXVIII and 38

C and 100

CCXXXIII and 233

IS THE HINDU-ARABIC NUMERAL ALWAYS EASIER TO WRITE THAN THE ROMAN NUMERAL?

Not always; see L and C examples above.

### SECTION 3 MEASURING WEIGHT

#### PURPOSE

- To develop the idea that weight is a property that can be used to describe and compare objects.
- To have the children discover that there are more precise methods of measuring the property of weight than by hefting.
- To have the children construct and use several devices for measuring weight and record the results in different ways.
- To review the idea that a fraction is a part of something, and to enlarge the children's understanding of the real number system by using fractions on the number line.
- To have the children recognize the value of standard units of measurement for the property of weight, as they did previously for the property of length.

#### COMMENTARY

In the seven lessons of this section the children learn more about measurement by studying the property of weight. In Lesson 19 they make and order gross comparisons of weight by hefting objects in their hands. Then the children improve their methods of measuring weight by constructing and using balances (Lessons 20 and 21). Variation in measurement is emphasized in Lesson 22 when the children develop two standard weight units to improve their measurements. In Lesson 23 fractional weight is related to the representation of numbers on the number line. In Lesson 24 the children construct another weighing device. Variation in weight measurement is demonstrated again in Lesson 25, when results obtained by different children are compared.

Lessons 22 and 25 should emphasize the fact that there is variation in all measurements. Variation is due, in part, to the inevitable errors which occur during the process of measuring. Independent measurements of the same quantity will

always differ, even though very slightly. Variation may also be due to differences in samples. Objects which are seemingly identical are never really identical and this fact also results in differing measurements of the same property. For example, one cork may look very much like another, but given sufficiently sensitive balances, one could show that each cork differs slightly in weight from all the others. Thus each object in the universe is unique, as we would find out if we could measure properties with sufficient precision.

Inaccuracy and variation are always involved in making measurements. By having the children work with weight, this section of the unit begins to illustrate these fundamental qualities of the measuring process.

## Lesson 19: A NEW PROPERTY

This lesson provides experience in ordering on the basis of a property other than length. The property of weight is introduced by having groups of children order objects by hefting them in their hands to judge the relative weights. Then each child is given a set of objects to order by hefting. He records his findings for this set on Worksheet 48.

Activity A is intended to give the children an opportunity to use their observational and descriptive powers to consider a great many properties before focusing on the property of weight. It is also intended as a review of recording with symbols. A segment of the Tor story is provided below. You may find it helpful in introducing the activities.

In Activity A give the children plenty of practice in recording with symbols on the chalk board before they do the worksheet in Activity B. Show them how the members of each set are represented by lowercase letters such as a, b, c, d, and that capital W represents the weight. Thus in ordering the members of Set A by weight, a child should make this symbolic record:

Set A (a, b, c)

a -- Ping-Pong ball  
b -- rubber ball  
c -- clay ball

$W_a < W_b$

$W_b < W_c$

$W_a < W_c$

$W_a < W_b < W_c$

The "greater than" symbol should also be used in the comparisons. The result for Set A:

$W_c > W_b > W_a$

If you want your class to review the concept of transitivity, this lesson provides a good opportunity. Simply ask the children, "When we know that the weight of a is less than the weight of b, and that the weight of b is less than the weight of c, is it necessary to compare the weights of a and c directly?"

## MATERIALS

For each group of 5 or 6 children, provide one of the Sets A-D. If you make substitutions, keep in mind that the three objects in a set should be similar in size and shape, but should vary considerably in weight.

- Set A { 1 Ring-Pong ball, 1 rubber ball, 1 ball of clay }
- Set B { 1 brick, 1 hard block, 1 soft block; or 1 hard block, 1 soft block, 1 closed box of similar size, etc. }
- Set C { 1 irregular stone, 1 papier-maché lump, 1 irregular piece of styrofoam }
- Set D { 3 covered paper cups or other containers filled with materials of three different densities } Select from such things as: soap powder, sand, sugar, salt, corks, small blocks, counters, etc.

-- for each child --

- set consisting of: { paper clip, crayon, reading book, scissors }
- Worksheet 48

## Activity A

If you wish to continue the story of Tor at this point, use the following or your own version of it.

### MORE ABOUT TOR

"Miss Johnson," Tor said, "I'm very happy about what I have learned about numbers and measuring with your class. I liked finding out about those ancient systems of numeration, too. And the number line, the addition slide rule, and the abacus were all very interesting. But I'm beginning to worry about two things."

"What things, Tor?" Miss Johnson asked.

"Well, the first thing is that I think I have forgotten how to use the 'greater than' and the 'less than' symbols."

"Oh, I'm sure the children will be willing to review those for you, Tor. We could start by comparing some objects for the property of length..."

"That reminds me of my second worry," Tor interrupted.

"Don't Earth People ever talk about any property but length? I'm really pretty tired of measuring length. There must be many other properties to consider..... maybe size, shape, color...?"

The children laughed. Tommy said, "I think we would all like to compare some other properties for a change." Then he picked up a Ping-Pong ball and bounced it. It bounced very high. Next he picked up a rubber ball. It bounced high, too, but not so high as the Ping-Pong ball. Finally, Tommy tried to bounce a ball of clay. It just went s-p-l-a-t and stayed on the floor.

"What property was I testing, Tor?" Tommy asked.

Tor shook his head. "I don't know," he admitted.

Sally said, "Tommy was comparing the bounciness of the three balls, weren't you, Tommy?"

"Say, that's more like it!" Tor said. "Now I'd like to hear my friends in this class describe and compare the properties of a great many objects."

"We will do better than that for you, Tor," Miss Johnson said. "The children will not only tell you how objects are alike -- they will also tell you some of the ways they are different."

"And, who knows? We may even find a very interesting new property to measure," Tommy said.

"Oh, that would be great!" Tor said.

Miss Johnson then brought out some sets of objects for the class to describe and compare. Tor was so curious about the objects that he leaned forward too far and nearly fell off his chair. Anyone could see that he could hardly wait for the lesson to start.

.....

Place enough Sets A-D around the room so that small groups of children can handle the objects and discuss the properties. Move from group to group, encouraging each group to discuss similar as well as different properties, including size, shape, color, softness, etc. (This can be either a group or a class discussion.)

Whether the children bring up the subject of weight themselves, or not, ask each group to determine how the weights of the objects compare by lifting (hefting) the objects in their hands. When each child has had an opportunity to heft at least one set of objects, have the class record the comparisons on the chalk board.

For a set of three objects, there are three binary comparisons which can be made and recorded:

$W_a < W_b$  (weight of Ping-Pong ball is less than weight of rubber ball)

$W_b < W_c$  (weight of rubber ball is less than weight of clay ball)

$W_a < W_c$  (weight of Ping-Pong ball is less than weight of clay ball)

Guide the children in recording the first set, but let them record the remaining sets on the chalk board. When this is done, ask the children to order each set:

$W_a < W_b < W_c$



Ask one child from each group to record the ordering of their set on the chalk board. Question the class about the relative weights of the objects shown in the ordering. Rotate groups, so that different children may check the ordering of each kind of set.

Now, if you wish to review or reinforce the concept of transitivity, ask the children whether it was necessary to compare the weight of object a with the weight of object c to determine the order of weight of three objects in a given set. From their experiences with measurements of length in first grade, some children should be able to explain that when they know that the weight of object a is less than the weight of object b, and that the weight of b is less than that of c, they do not need to compare a and c directly, because they can predict that the weight of a will be the least and the weight of c the greatest. Therefore only two binary comparisons were really needed.

#### Activity-B

See that each child has a paper clip, a crayon, a reading book, and scissors on his desk. Ask the class to turn to Worksheet 48. Explain how the symbols are used and what the children are to do to complete this worksheet. Instruct the children to heft the objects two at a time and arrange them in order by weight on their desks -- the object of least weight at the left. Each child should record the six binary comparisons required by the worksheet. Then he should order his data on the last line.

Worksheet 48  
Unit 16

Name \_\_\_\_\_

#### Symbol

$W_a$  = Weight of the paper clip  
 $W_b$  = Weight of the book  
 $W_c$  = Weight of the scissors  
 $W_d$  = Weight of the crayon

Compare the weight of your objects by picking up one in each hand. Then write the proper symbol  $<$ ,  $>$ , or  $=$  in each blank.

$W_a$   $<$   $W_c$

$W_b$   $>$   $W_c$

$W_c$   $>$   $W_d$

$W_a$   $<$   $W_b$

$W_a$   $<$   $W_d$

$W_b$   $>$   $W_d$

Compare the weights of all four objects. Write the letters for the objects in the blanks.

$W_a < W_d < W_c < W_b$

Again transitivity may be explored by asking whether it was necessary to make all six binary comparisons in order to fill in the last line of the worksheet. Some of the children may be able to tell you that sometimes only three binary comparisons will actually be needed to determine the order of weight for the four objects.

You may wish to conclude the lesson by saying that you hope Tor likes the new property -- weight -- which the children have been comparing, and that in the next lessons they will be considering some ways, other than hefting, for comparing this property.

## Lesson 20: USING A SEESAW

The idea that a beam balance is useful for comparing the weights of two objects is developed in this lesson. In Activity A the children are challenged to find a method of comparing the weight of a 5-unit Minnebar with that of a paper cup by means of a ruler balanced on a pencil.

In Activity B, a seesaw is used to order all the children in a set by weight. It may not be practical for you to include all the children in the class in this set. Determine how many children you can conveniently include in the set after reading the activity. If some children are sensitive about their weight, due to obesity or extreme thinness, assign them to help you with the equipment, keep the class in order, record data, etc.

### MATERIALS

-- for each child --

- pencil
- ruler
- 5-unit Minnebar
- paper cup

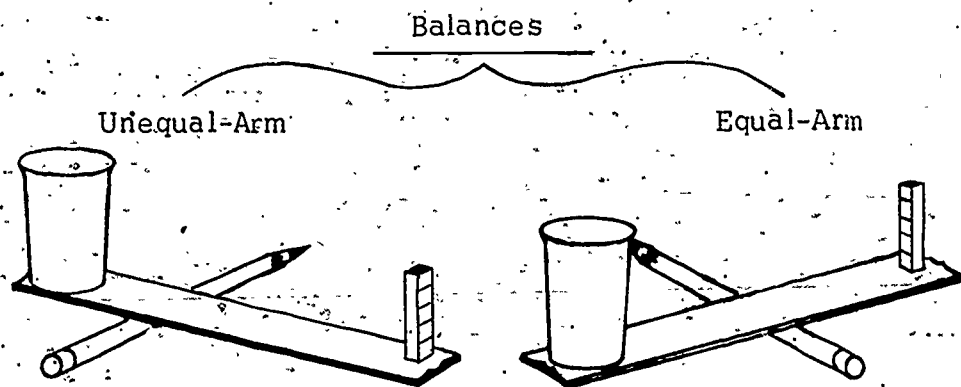
-- for the class --

- seesaw (or 6' x 8" x 2" plank balanced on a brick)

### PROCEDURE

#### Activity A

See that each child has on his desk a pencil, ruler, 5-unit Minnebar and a paper cup. Challenge the children to discover ways to use these objects to decide whether the Minnebar is heavier or lighter than the cup. Most of the children will devise arm balances. Some may be equal-arm balances, and some may be unequal-arm balances, as shown in the diagram on the next page.



Permit students to use and observe their balances before you discuss the merits of one or the other. Invite the children to show any method of comparing objects by weight which is different from the equal-arm balance method. Then ask:

WHAT IS THE DIFFERENCE BETWEEN AN EQUAL-ARM BALANCE AND AN UNEQUAL-ARM BALANCE? (The pencil is centered beneath the ruler for the equal-arm type.)

Have the children use their balances to answer the next two questions:

HOW CAN YOU TELL WHICH OBJECT IS HEAVIER THAN ANOTHER IF YOU ARE COMPARING THEIR WEIGHT WITH AN EQUAL-ARM BALANCE? (The heavier object makes its end of the ruler go down.)

HOW CAN YOU TELL WHICH OF TWO OBJECTS IS HEAVIER WHEN YOU ARE USING AN UNEQUAL-ARM BALANCE? (It is difficult, but you can tell; at balance, the shorter arm carries the greater weight.)

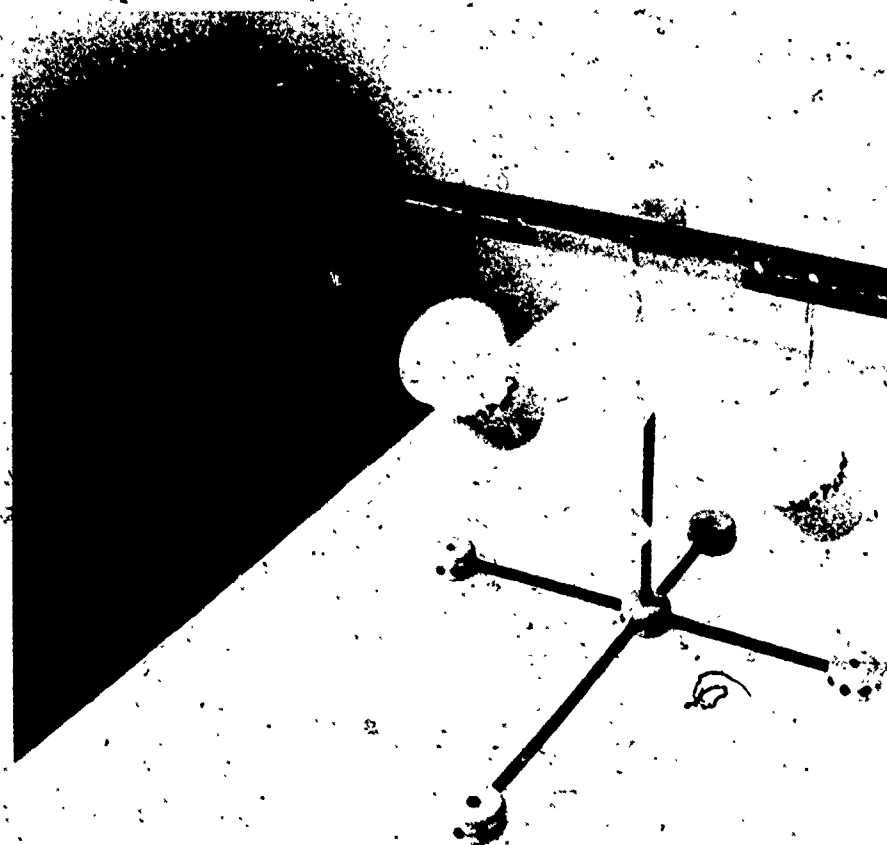
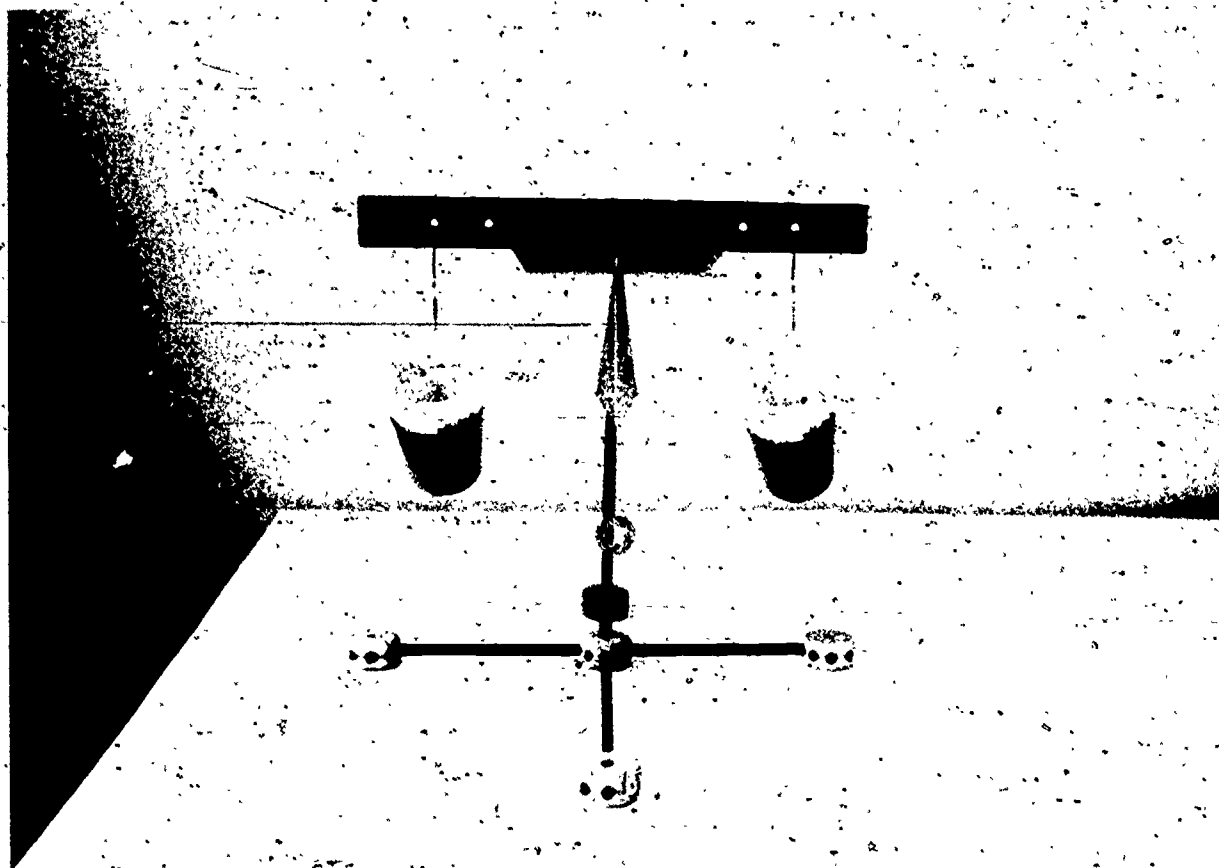
#### Activity B

Select a set of children to be weighed. Then ask the class how you could arrange this set of children in order according to their weight. If they suggest lifting, say that this would be too difficult for you -- you want an easier method. If the idea of using an equal-arm balance does not come from the class, ask if a seesaw or teeter-totter might be useful. When this has been discussed, use a seesaw (or the plank and brick mentioned in the materials) to do the ordering.

Several seesaws in the playground could be used, if they are available and if the children will remain fairly orderly under a classmate's supervision.

As the first step in ordering the set of children by weight, select one child as the standard and sort all the children into three subsets -- the children with more weight than the standard, the children with less weight, and the children with about the same weight.

If you selected a very large set of children (or the entire class), it will be necessary to select a child in each subset as the standard for that group, and sort the members of the subset in the same way, until all the children have been arranged in order by weight.



## Lesson 21: BUILDING A BEAM BALANCE

This lesson familiarizes the children with the construction and use of a simple beam balance, and makes them realize the need for a standard unit of weight. After using this beam balance, the children see the advantages it has over the seesaw they used in Lesson 20.

Small groups of children construct the balances, then use them to compare the weight of two objects or of combinations of objects. It is essential that you construct a beam balance before class, using the instructions and photographs given in the lesson. This will help you select the tasks where the children need the most help, and will provide a model for them to copy. Allow one full class period for the children to assemble their balances.



## MATERIALS

- 1 assembled beam balance.
  - for each group --
- 1 balance kit containing: 1 ruler, 2 paper cups, 1 washer, 12" piece of thread, cardboard triangle, 2 paper clips, 10" strip of masking tape, picture hook, 2 red slotted (5") Tinkertoy rods, 1 blue slotted ( $3\frac{3}{8}$ " ) rod, 1 green slotted ( $7\frac{1}{4}$ " ) rod, 1 purple slotted ( $10\frac{1}{16}$ " ) rod, 6 round Tinkertoy connectors
- red rubber balls
- Ping-Pong balls
- pencils
- scissors
- paper cups
- corks
- small paper clips,  $1\frac{1}{4}$ " long

## PROCEDURE

### Activity A

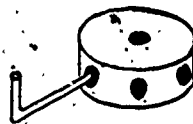
Place an assembled equal-arm balance on your desk, and drop a couple of weights into the cups. Let the children take a minute or two to look at it, and tell them that they are going to construct this balance and experiment with it.

Divide the class into groups of three or four and give one balance kit to each group. Have the children assemble their balances while you give instructions step by step. Draw diagrams on the chalk board and give help as needed.

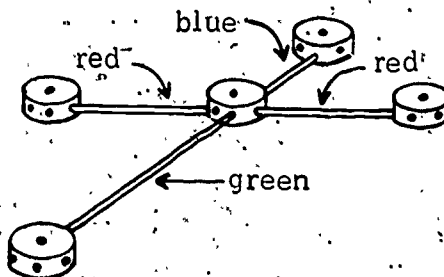


### Step I. Assembling the Tinkertoy Stand

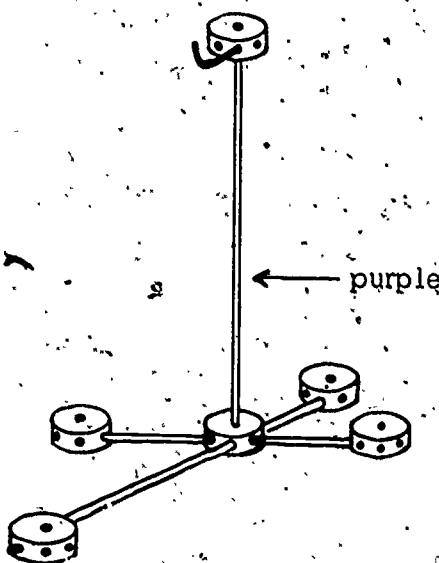
1. Screw a picture hook into one of the holes of a round connector and put it aside.



2. Assemble the base of the stand, using 5 round connectors, 1 blue rod, 2 red rods, and 1 green rod. (Have the children look at the base of your model and assemble theirs in the same way.)

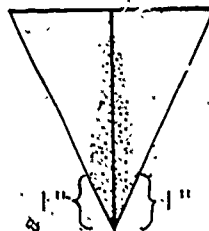


3. Insert a purple rod in the center connector of the base. Place the round connector with the picture hook on the other end of the purple rod. Your stand assembly is now complete.

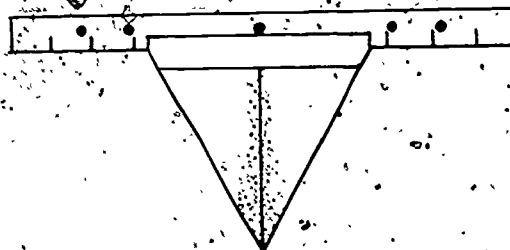


## Step II. Assembling the Beam

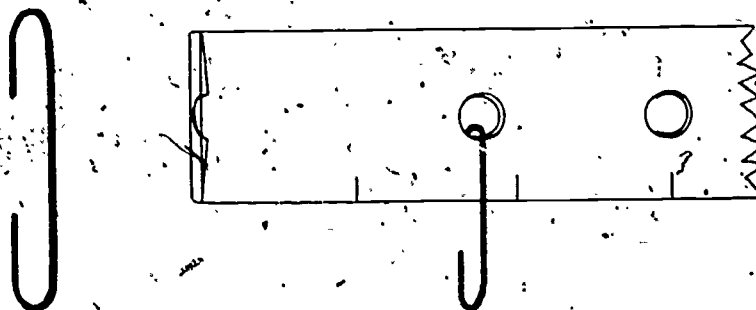
1. Take your cardboard triangle and measure up one inch on each side from the bottom. Draw a line from each of these points to the top of the center line. Color this diamond-shaped area red. (You may have to draw the lines for the children yourself and let them do the coloring.)



2. Using masking tape on both front and back sides, attach the cardboard triangle to the edge of the ruler so that the line on the triangle is centered at the hole in the middle of the ruler (i.e., line up the line on the triangle at the 6" mark).



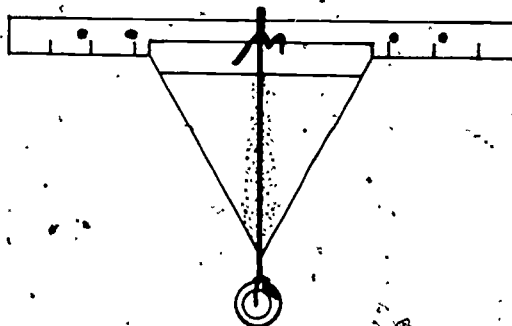
3. Unbend paper clips to make two hooks and insert one in each of the holes near the ends of the ruler.



4. Mount the balance arm on the stand by slipping the center hole of the ruler over the picture hook.

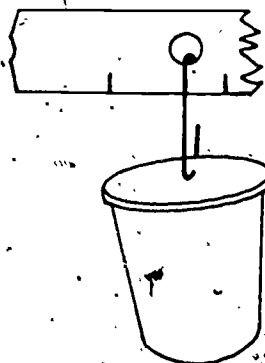
### Step III. Making the Bob

1. Cut a piece of thread or thin string about 12" long.
2. Tie a washer to one end of the string.
3. Make a small loop at the other end of the string.
4. Slip the loop of the bob assembly over the picture hook.



### Step IV. Attaching the Cups and Adjusting the Balance

1. Punch a small hole under the rim of each of the two cups and hang the cups on the paper-clip hooks attached to the balance arm. (Be careful to attach the cup as shown in the diagram; this lessens the chances of the children scratching their hands on the hooks when they use the balance.)



2. If your balance needs adjustment to hang evenly, take a piece of masking tape 1" long. Move it around on the balance arm until balance is obtained. Stick the tape on the back side of the balance arm at this point. (You may need to give the children considerable help with this.)

### Activity B--

Let the children experiment with the beam balances. Have them compare the weights of objects they compared in Lesson 20 when using the ruler balance. On the chalk board, record some of these comparisons. If possible, use the data to predict other relations of the weights of objects (transitivity). These questions will help with the discussion:

WHICH OBJECT WEIGHS LESS THAN THE PENCIL?

WHICH OBJECT WEIGHS LESS THAN THE PING-PONG BALL?

WHICH OBJECT WEIGHS THE LEAST?

After the children have had time to experiment, ask them to discuss the advantages the beam balance has over the ruler balance. The discussion should include questions that point to the following advantages of the beam balance:

1. The cups are convenient for holding objects to be weighed.
2. It is easy to decide when the weights balance.
3. It is more reliable than the ruler balance because it can be adjusted to balance evenly -- with tape, when not loaded.
4. The arms are equal in length.
5. It shows smaller differences in weight.

NOTE: The assembled beam balances are also used in Lessons 22 and 25.

## Lesson 22: STANDARD WEIGHT AND VARIATION

The desirability of using a standard unit of weight is introduced in this lesson. The idea is projected by having various groups of children determine the weight of similar rubber balls by using different objects as units of weight. The children will then explore the reasons why their weight measurements vary. An attempt will be made to develop the idea of a two-unit weight standard.

### MATERIALS

-- for each group --

- 1 assembled beam balance from Lesson 21
- 1 rubber ball

--- for the various groups ---

- several Ping-Pong balls (Group 1)
- several pairs of scissors (Group 2)
- 4 paper cups (Group 3)
- 15 small paper clips, 1" long (Group 4)
- 5 large paper clips, 2" long (Group 5)
- 15 corks (Group 6)

-- for the class demonstration --

- 1 piece of chalk, about 1" long
- 1 small rubber eraser, about 1" long

-- for each child --

- Worksheet 49

### PROCEDURE

#### Activity A

Divide the class into six groups of children. See that each group has an assembled beam balance, a rubber ball, and one set of the objects listed in the materials. Tell the children they are going to carry on an investigation to help the class decide on a standard unit of weight. The object to be weighed is the rubber ball.

Ask the children to weigh the ball with the beam balance, using the set of similar objects they were given as units of weight. Record on your chalk board the different combinations of weight that the different groups find. Remind the children that they are doing an investigation for the class and that they should observe the advantages and disadvantages of the units of weight they are using. Rotate your objects so that different groups will have the opportunity to weigh the ball using different objects as units of weight. After the children have had time to investigate these units, discuss the advantages and disadvantages of each weight unit, i.e., not enough room in the cup, takes too many units, never quite balances, etc. After several of these problems have been discussed, ask the children what can be done to improve their units of weight.

LET'S TAKE TWO NEW OBJECTS AND WEIGH THEM USING PAPER CUPS THE FIRST TIME, SMALL PAPER CLIPS THE SECOND TIME. ONE OF YOU WILL RECORD THE RESULTS ON THE CHALK BOARD CHART.

<u>Objects</u>	<u>Number of Paper Cups</u>	<u>Number of Small Paper Clips</u>
Chalk	0-1 (can't weigh)	5
Eraser	2-3	22

WHAT CAN WE LEARN FROM THIS CHART? (We use fewer cups than clips. A cup is too heavy to balance a very light object.)

WHAT UNITS OF WEIGHT DOES THE SCHOOL NURSE USE TO WEIGH YOU? (Pounds and ounces.)

WHY DOES SHE USE TWO UNITS OF WEIGHT? (To get closer to the actual weight.)

## Activity B

Divide the class into groups of three. Remind the children that they have found out that they can weigh objects with one unit but not with another. Ask each group to use beam balances to compare the weights of paper cups, large paper clips, small paper clips, and corks. Then have the children turn to Worksheet 49 and fill in the upper half. This worksheet should help them discover some facts that will eventually assist them in developing a two-unit standard of weight. Answers from each group may vary considerably.

Instruct the children that for our purposes, a reading may be taken on the beam balance when the red area is behind the white thread. Tell the children to start their investigations and record their data on the worksheet.

After a few minutes ask a child to read the answers on his worksheet. You may ask:

DOES EVERYONE AGREE? (No, read more answers.)

Keep in mind that we told the children that a reading is taken within the red area. Therefore, they can obtain the weight by reading on either side of the center line. Have each group weigh one paper cup using small paper clips. For one paper cup, you might get a reading of nine small paper clips or one of ten small paper clips. After the children have finished setting up their equipment have a team of three students read and record the weight of the cup for each beam balance. Record

Worksheet 49		Name _____	
Unit 16			
Use your beam balance, paper clips, corks, and cup to help you complete the following.			
1 paper cup	≅	<input type="text" value="10"/>	small paper clips
1 paper cup	≅	<input type="text" value="16"/>	cork stoppers
1 large paper clip	≅	<input type="text" value="5"/>	cork stoppers
1 large paper clip	≅	<input type="text" value="3"/>	small paper clips
* 1 paper cup	≅	<input type="text" value="3-4"/>	large paper clips
* 1 paper cup	≅	3 large paper clips +	<input type="text" value="1"/> small paper clips

these weights on the chalk board. Ask the teams to explain to the class why the measurements are different. (Different sections of the red area were behind the center thread.)

Pick two teams to weigh a piece of chalk using small paper clips as the standard weight unit. The class will observe the operation so that they can answer the following questions:

HOW MANY STUDENTS OBSERVED THE SAME READING  
ON BOTH EQUAL-ARM BALANCES?

HOW MANY HAD DIFFERENT READINGS?

Ask each group to look at the center thread from three positions. The first at the left of the thread, the second in front of the thread and the third at the right of the thread. Spend a few minutes talking about these questions:

WHY DO READINGS VARY?

WOULD THE DIFFERENT READINGS BE CORRECT FOR  
OUR PURPOSES?

DO MOST MEASUREMENTS VARY?

Discuss the idea that when a measurement is made several times, we may obtain a different answer each time because of measuring error.

As a class activity, help the children complete the lower part of Worksheet 49. (The items are starred because they are difficult.) You may wish to discuss the fact that in the first example, 3 large paper clips do not weigh enough and 4 weigh too much, therefore, it is difficult to measure the weight with large paper clips. Now, 3 large paper clips plus 1 or 2 small clips is considerably closer to the weight of 1 paper cup, therefore, a better measurement.



## Lesson 23: MEASURING CHILDREN'S WEIGHTS

In this lesson the children are weighed on a bath or nurse's scale. The children should assist in weighing other children, in reading the scale, and in recording the weights on the chalk board. From the data collected, the children make a histogram which graphically demonstrates the variation and distribution of their weights.

As in Lesson 20, keep in mind that some children may be sensitive about their weights. Let these children help with the tasks of weighing and recording.

### MATERIALS

- bath or nurse's scale for weighing the children
- Worksheet 50
- overhead projector (optional)

### PROCEDURE

#### Activity

Use a scale to weigh the children in your class. Ask a child to record the weights on the chalk board. This first record should be a simple one -- just a list of the weights as they are read, e.g:

43, 47, 39, 45, etc....

Have the children weigh each other, too, so that they get practice in reading the scale.

When all the weights are written on the chalk board, involve the children in a discussion of the data recorded there. Ask what the highest and lowest recorded weights are, etc. (This information will be helpful when the children make the histogram.)

After the discussion, rewrite the weight data in such a way that all similar weights are grouped together:

47, 47, 47

52, 52

43

45, 45, 45, etc....

Ask the children to turn to Worksheet 50 and place it on their desks so that one line is at the left and the other line is at the bottom of the paper. Tell the children to think of the bottom line as a number line where each space will represent a weight range of three pounds. Have them look at the lowest recorded weight on the chalk board and start numbering the intervals with a value that is three pounds less than that. Then they should fill in all the intervals, going to the right along the bottom. For example, if the lowest weight recorded for any child is 33 pounds, mark the first space under the number line as follows:

30
to
32

HISTOGRAM														
NUMBER OF CHILDREN	10													
	9													
	8													
	7													
	6													
	5													
	4													
	3													
	2													
	1													
		to	to	to	to	to	to	to	to	to	to	to	to	to
WEIGHT IN POUNDS														

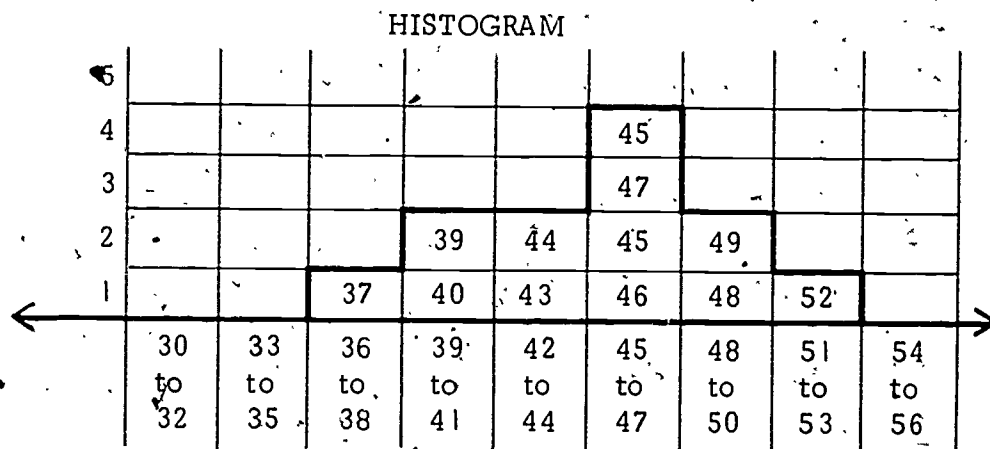
Worksheet 50  
 Unit 16  
 Name \_\_\_\_\_

In this case the spaces below the number line would read like this:

30	33	36	39	42	45	48	51	54	57	60	63	66	69
to	to	to	to	to	to	to	to	to	to	to	to	to	to
32	35	38	41	44	47	50	53	56	59	62	65	68	71

Demonstrate the procedure on the chalk board or overhead projector. Then ask the children to listen carefully as you read the list of weights from the chalk board and to write each number, as you give it, in the first empty square above the appropriate place on the number line.

When the children have made all their entries, have them check the accuracy of their work with your own histogram, either projected or on the chalk board. Then show them how to draw heavy lines to enclose all the squares they have filled-in:



Discuss with the class the labeling of the horizontal and vertical axes and why it is important. (It tells the number of children in each weight group.) Ask the class to help you decide on a title for the histogram.

Have the children study their histograms and try to answer such questions as:

HOW MANY CHILDREN WEIGHED BETWEEN 39 AND 41 POUNDS?

## HOW MANY MEMBERS ARE THERE IN THE SET OF CHILDREN WEIGHING 48 TO 50 POUNDS?

Try to get the class to think about the variation in weights that has been observed. Ask whether the children expected that all in the class would weigh the same. Ask about their heights and whether there is a variation in height similar to that in weight. Point out that there are no two things exactly alike in the world, and that there are always some differences among samples. Even things that look alike have very slight differences. If the lengths of two new unsharpened pencils were measured very carefully, the measurements would be slightly different.

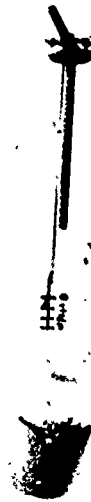
The fact that samples vary, and the impossibility of measuring things exactly (as observed in the last lesson), are the two main contributions to the children's understanding of variation.

## Lesson 24: CONSTRUCTING A RUBBER-BAND SCALE

In this lesson the class develops a new weighing device, using the stand from the beam balance made in Lesson 21. Working in groups, the children build, calibrate, and use a rubber-band scale to weigh various objects. The rubber band (an elastic string) is the "spring" of the new weighing scale.

Calibrating is assigning to each point on a number line a value corresponding to a standard unit of measurement. In this lesson the units used for calibrating are paper cups and the calibrating is done vertically from top to bottom. When only one paper cup is hanging from the rubber band, the children notice where the pointer is, make a mark, and label that mark zero. As they add more cups, they make other marks and numerals until they have marked units of weight from zero to ten.

Be sure to construct a rubber band scale in advance, so that the children can look at it while building their own. Follow the four steps given in the lesson. When calibrating, notice that the marks will be increasingly farther apart, as more cups are added. This is to be expected, as the rubber string stretches more as the load increases.



## MATERIALS

- Worksheet 51
- small paper clips
  - for each group --
- beam balance from Lesson 21
- a few Minnebars, other objects from desks
- 2" x 14" strip of oak tag
- large paper clip, 2" long; 1 rubber or elastic string
- 8" green Tinkertoy rod, 3½" blue Tinkertoy rod, cylindrical Tinkertoy connector
- 11 paper cups

## PROCEDURE

### Activity

Ask a child to use the beam balance from Lesson 21 to weigh some small paper clips, using paper cups as his weight units. Make a record of several such measurements on the chalk board to review the comparison symbols: less than (<), greater than (>), and appears to be the same as ( $\doteq$ ), for the class, e.g:

Weight of 1 paper cup < Weight of 11 small clips < Weight of 2 paper cups

or

Weight of 2 paper cups > Weight of 11 small clips > Weight of 1 paper cup

and

Weight of 1 paper cup  $\doteq$  Weight of 10 small clips

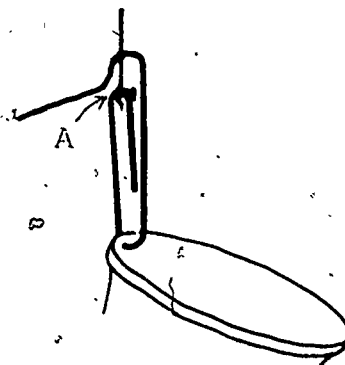
After this review of the comparison symbols, say that Tor wonders whether all weighing scales are like the beam balance just used. Ask the children to tell Tor about other weighing scales they have seen. (Some children may have noticed spring scales being used to weigh produce; others may tell about penny weighing machines; etc.) Then hold up an elastic string, and ask the children whether they think this

elastic string could serve as the spring for a new kind of scale. Let the children think about and discuss this. Then divide the class into groups such that each has a beam balance from Lesson 21, and distribute the rest of the new equipment.

Use the instructions inside the four boxes to help the children make their rubber-band scales. Draw on the chalk board any diagrams that are necessary to clarify the process. Also, remind the children that they may come up and look at the rubber-band scale you have already made.

Instructions for the rubber-band scale:

Step 1. Take the large paper clip and bend one end up. Connect the clip to a paper cup by pushing the same end of the clip through the wall of the paper cup just below the rim. Tie the rubber band at point A. Tie the other end of the rubber band to the blue rod.



When the children have done this, ask each group to put a Minnebar in the paper cup. Ask what happened when they put the Minnebar in the cup. (The rubber band stretched and the cup went down.) Next ask whether this device is a good scale for weighing. Discuss and record suggestions given by the children. Give them a few minutes to experiment. Tell them to raise and lower the blue rod and see what happens. Then ask:

IS THIS EQUIPMENT YOU CAN HOLD IN YOUR HAND A GOOD WEIGHING SCALE? IS IT A POOR ONE? WHY DO YOU THINK SO? HOW CAN WE IMPROVE IT?

The children may suggest that the device they have made is easy to carry around; that it has no scale so that it is impossible to tell how many units an object weighs; that it ought to hang from a stand, etc. Elicit as many suggestions as possible; then ask whether any of this information could help in developing a new scale. After discussion, go on to the second instruction for making the rubber-band scale.

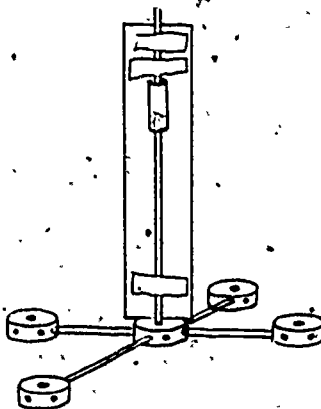
Step 2. Remove the ruler with its two cups, etc., from the purple rod of the beam balance. Place the cylindrical connector on the purple rod, then place a green rod in the top end of the cylinder. Next put the round connector on top of the green rod, so that later, when it is wanted, the balance scale may be put on the back of the stand again. Then put the blue rod in the round connector so that it is located on the front of the stand.

Have the children weigh a few objects and ask which object weighed the most. Point out that it is difficult to remember how far the rubber band stretched or how far down the cup moved. Then ask:

WHAT CAN WE DO THAT WILL MAKE IT EASIER FOR US  
TO REMEMBER WHAT THE DIFFERENT OBJECTS WEIGH?

After discussion, have the children proceed to the next step of the construction. Give whatever help is needed.

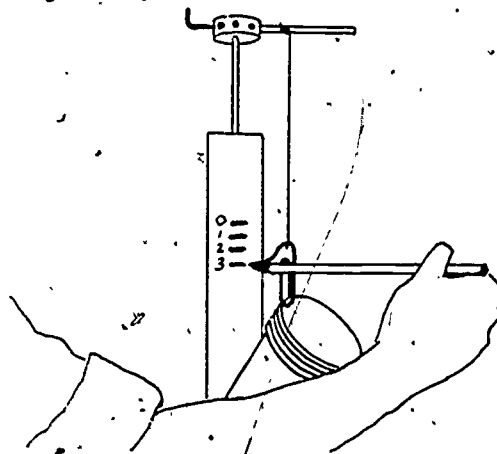
Step 3. Use masking tape to fasten the oak tag strip to the upright rod. Place it so that the bottom of the oak tag rests on the round connector at the bottom.





Discuss with the children how they can make a scale on the oak tag so that objects can be weighed. Ask whether paper cups could be used as standard weight units. Finally, explain that you want the children to calibrate the rubber-band scale with paper cups as the standard units of measurement. Demonstrate what you mean by calibrating during the next step of the construction.

Step 4. Look at the turned-up end of the large paper clip. This is the pointer. Place a pencil beside the pointer and make a mark on the oak tag. Label the mark 0. Now place another paper cup inside the cup at the bottom of the rubber band. Place the pencil beside the paper-clip pointer again and mark the oak tag at this new place. Label this second mark 1. Continue adding a cup at a time to the other cups, making a new mark at each place where the pointer stops until you have marks and numerals on the oak tag from 0 through 10.



During the calibrating, some children may notice that the marks are not the same distance apart. If they cannot account for this, explain that the rubber band stretched more as the load of paper cups increased.

When the new weighing scales are ready, have the children select objects they would like to weigh and record the weights on Worksheet 5. Encourage the children to experiment with many kinds of objects to discover that not all objects can be weighed on the rubber-band scale. Ask the children to put the beam balance equipment on the back of the rubber-band scale and weigh some objects on both the beam balance and the rubber-band scale.

Worksheet 51  
Unit 16

Name \_\_\_\_\_

Object	Weight
A _____	_____
B _____	_____
C _____	_____
D _____	_____
E _____	_____

Fill in the blanks with  $<$ ,  $>$ , or  $=$ .

1. Weight of object A \_\_\_\_\_ Weight of object B.
2. Weight of object C \_\_\_\_\_ Weight of object D.
3. Weight of object E \_\_\_\_\_ Weight of object A.
4. Weight of object D \_\_\_\_\_ Weight of object B.
5. Weight of objects A and B \_\_\_\_\_ Weight of objects D and E.

Conclude the lesson by asking the children to discuss for the advantages and disadvantages of the rubber-band scale. Children will probably respond by saying that it is convenient to be able to read the weight directly from the scale. A few may notice that the rubber band becomes stretched from continued use, so that the calibrations are not as accurate as they were at first.

## Lesson 25: EXPERIMENTS WITH A BEAM BALANCE

In this lesson the children do experiments with their beam balances to find out which factors may or may not affect the weight of various objects. They select questions from Worksheets 52 and 53, or other questions of yours or of their own. All questions should require a simple experiment with the beam balance. The children work in groups, each group selecting a different question and deciding on the procedure to be used to answer the question. Each experiment involves weighing an object before and after its weight has, or has not, changed (with time, for example).

Begin the lesson with a demonstration that answers the question, "Will this birthday candle weigh more, less, or about the same, if I burn it for a while?" Have the children make predictions and tell you how to go about answering the question. Ask them to suggest a standard weight for you to use and to show you how to record the before-and-after weights, etc.

Some of the questions on the worksheets may not be practical for your class; finding substitute questions will afford the children good opportunities to devise experiments with whatever materials are available.

### MATERIALS

- Worksheets 52 and 53
- birthday candle, matches
  - for each group --
- beam balance
- small paper clips,  $1\frac{1}{4}$ " long
- paper cups
- one selection at a time from the following materials (or substitutes): ice cubes; cotton; water; apple; sand; eggs; spring or elastic; clay; milk; pencil, chalk or crayon; bar of soap

## PROCEDURE

### Activity A

Suggest that Tor has been very interested in the beam balances. He thinks that the children could have fun with the balances if they used them to answer some of his many questions. Say that the first question he brought up was about a candle:

TOR WANTS TO KNOW IF BURNING A CANDLE WILL CAUSE IT TO LOSE WEIGHT, GAIN WEIGHT, OR STAY AT THE SAME WEIGHT. HOW CAN I ANSWER HIS QUESTION?

Show a birthday candle to the class and ask what you should do to solve the problem. Help the class develop ideas for the experiment. As the following ideas are presented, carry them out for the children.

1. Weigh the birthday candle, using small paper clips as the standard weight units. Record the weight on the chalk board.
2. In a safe place, light the candle with a match.
3. Let the candle burn until it is about one inch shorter.
4. Weigh the candle again. Record the new weight.

If the children are interested, let them discuss why the candle weighed less after burning than before. (The particles changed form and passed into the air.)

### Activity B

Divide the class into groups, each with a beam balance. Have each group select a problem from Worksheets 52 or 53. Instruct the children to use either paper clips or paper cups as their standard weight units. Assist the various groups in trying to develop methods for answering the questions. Show them where the equipment is that you have provided. Move from group to group, giving suggestions that will help the children get started. Let each group perform several experiments. Give as much help as is needed, but if you feel that some of the experiments will take too long or are impractical for your class, substitute others.

After a group has finished an experiment, encourage the children to discuss the results. Leave a few beam balances in a convenient place for those experiments which require evaporation over a number of days, etc.

Worksheet 52

Unit 16

Name \_\_\_\_\_



Tor wants to know:

1. Will the weight of an ice cube change after it has melted? (Weigh the ice cube in a cup and weigh the water in the cup and weigh again.)
2. Do two cups of cotton weigh the same? (Have two cups of cotton. Fill several cups with cotton. Weigh them.)
3. Will stretching a spring or rubber band change its weight? (Weigh the spring or elastic band. Weigh it after you stretch it. Add the weights. Stretch the spring or elastic band with the pencil or ruler. Weigh.)
4. Will the weight of a piece of clay change if we divide it into smaller pieces? (Weigh the whole piece of clay. Divide it into several pieces. Weigh each piece. Add the weights. Weigh the whole piece of clay together again and weigh again.)
5. Will the weight of a pencil, a crayon, or a piece of chalk change after it is used? (Weigh a new pencil, crayon, or piece of chalk. Use it. Weigh again.)
6. Does a cup of wet sand weigh more than a cup of dry sand? (Weigh a cup of each.)

Worksheet 53

Unit 16

Name \_\_\_\_\_



Tor wants to know:

7. Will the weight of a bar of soap change after using it? (Weigh a new bar of soap, use and weigh again.)
8. Will an apple lose weight sitting on the teacher's desk? (Weigh an apple. Let it sit for 4-5 days, weigh again.)
9. Will the weight of a cup of water change if it is allowed to sit in the classroom for a few days? Will covering the cup make any difference? (Weigh two cups of water. Cover one and let both cups sit for 2-3 days. Weigh again.)
10. Will the weight of a cup of water change after boiling the water? (Weigh the water. Boil for 10 minutes. Weigh again.)
11. Will a hard-boiled egg weigh more than a soft-boiled egg? (Weigh the eggs. Boil one for 10 minutes, another for 2 minutes. Cool the boiled eggs and weigh each of the eggs again.)
12. Will a carton of milk weigh the same after it turns sour? (Weigh the carton of milk before and after the milk sours.)

## SECTION 4 NEGATIVE NUMBERS

### PURPOSE

- To provide practice in directed movements along the number line.
- To introduce negative integers.

### COMMENTARY

The experiences gained in this section expand the usefulness of the number line. In these two lessons direction of movement along the number line is considered in order to help explain negative numbers. The number line is shown as extending infinitely to the left as well as to the right.

## Lesson 26: DIRECTIONS ON THE NUMBER LINE

This lesson gives the children practice in directed motion along a number line. It prepares for the introduction of negative numbers in the next lesson.

### MATERIALS

= Worksheets 54 and 55

### PROCEDURE

#### Activity A

Suggest to the class that Tor has noticed that many measurements -- including those of weight -- involve reading scales of numbers that are like the number line. He has asked if there are more ways of using the number line, and if there is more he should know about numbers.

Draw a number line on the chalk board. (Remember to put an arrow at each end of the number line.) Label points from 0 to 16. Then mark an X at point 8. Ask the children to tell you at what point you will be if you start at X and move 3 spaces to the right. (11.) Record this move on the board as:

Start at X and move  $\overrightarrow{3}$

Explain that the arrow over the numeral shows the direction of the move.

Start at X again, but this time move 5 to the left. Record this on the board as:

Start at X and move  $\overleftarrow{5}$

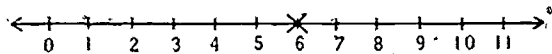
Ask a child to explain the symbolism. Go through several more examples of moves to the right and moves to the left, but be sure not to end up to the left of 0.

When the children are familiar with the procedure, have them complete Worksheet 54. Remind them that they can always check their answers on the number line provided on the worksheet.

Worksheet 54  
Unit 16

Name \_\_\_\_\_

Start at X. Write a numeral in the box to show where you are after each move.

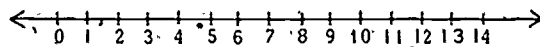


Move 4 $\rightarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">10</div>	Move 5 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">1</div>
Move 3 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">3</div>	Move 3 $\rightarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">9</div>
Move 0 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">6</div>	Move 1 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">5</div>
Move 2 $\rightarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">8</div>	Move 5 $\rightarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">11</div>
Move 6 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">0</div>	Move 2 $\leftarrow$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">4</div>

Worksheet 55  
Unit 16

Name \_\_\_\_\_

Write a numeral in each box to show where you are after the move shown. Use the number line to help you find the answer.



$8 + 4$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">4</div>	$10 + 3$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">7</div>
$6 + 3$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">9</div>	$2 + 5$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">7</div>
$7 + 5$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">2</div>	$1 + 11$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">12</div>
$13 + 7$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">6</div>	$5 + 5$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">0</div>
$0 + 11$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">11</div>	$6 + 4$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">10</div>
$6 + 0$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">6</div>	$11 + 1$	$=$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">12</div>

Activity B

Show the children that if they did many problems starting at different points, the number lines would be full of X's. Say that they can avoid this by writing the numbers that indicate where they start on a piece of paper. For example, if they want to start at 8, they could just write 8. To show that this is followed by 3 moves to the right, they could write  $8 + 3$ . Demonstrate several examples on the chalk board:

$\rightarrow$   
 $8 + 4$  (8 followed by 4 moves to the right)

$\leftarrow$   
 $7 + 6$  (7 followed by 6 moves to the left)

$\rightarrow$   
 $3 + 2$  (3 followed by 2 moves to the right)

Ask the children to do Worksheet 55.



## Lesson 27: NEGATIVE INTEGERS

This lesson introduces negative integers to the children by extending the number line to the left of zero.

### MATERIALS

- desk number line (-15 to +25) for each child
- Worksheets 56, 57, and 58

### PROCEDURE

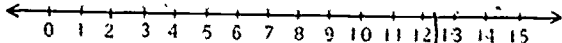
#### Activity A

Draw a number line on the chalk board. Label it from 0 to 15, leaving some space to the left of 0. Ask the children if a move of  $8 + 3$  can be represented simply by  $8 + 3$  and if  $4 + 6$  can be written simply as  $4 + 6$ . (Yes.)

Now raise the question of how we could represent  $6 + 3$ . If no child suggests it, show the class that  $6 - 3$  is the same as  $6 + 3$ . Repeat with  $7 - 2$  for  $7 + 2$ , etc. Work several examples on the chalk board number line, such as  $8 - 7$ ,  $7 - 4$ ,  $6 - 2$ ,  $6 - 5$ .

Worksheet 56  
Unit 16 Name: \_\_\_\_\_

Use the number line to help you find the answers.



$5 + 2 =$	<div>7</div>	$2 + 9 =$	<div>11</div>
$6 + 3 =$	<div>9</div>	$9 - 2 =$	<div>7</div>
$7 - 7 =$	<div>0</div>	$4 + 6 =$	<div>10</div>
$9 - 3 =$	<div>6</div>	$9 - 9 =$	<div>0</div>
$12 + 2 =$	<div>14</div>	$12 - 5 =$	<div>7</div>

Now ask the children to give you another name for  $6 - 3$ . (3.)

Have one child go to the chalk board and go through the steps in calculating  $6 - 3$ . Try this with  $5 - 3$ ,  $4 - 3$ ,  $3 - 3$ . In the last case, show the class that you arrive at 0 by moving 3 steps to the left of 3.

Have the children complete Worksheet 56.

Now do  $2 - 3$  at the chalk board. Try to move 3 spaces to the left of 2 on the number line.

WHAT CAN WE DO IN ORDER TO SHOW THIS MOVE?  
(Mark units on the number line to the left of 0.)

DO WE HAVE A NAME FOR THIS LOCATION ON THE NUMBER LINE? (Not yet.)

To find the name for this point ask how many units to the left of 0 it is. (1.) Explain that since we represent movement to the left with a minus-sign, we call the point just to the left of 0, "minus one" (-1). Label the point.

Show examples such as  $2 - 4$ ,  $4 - 7$ ,  $9 - 13$ , and  $6 - 11$ , to generate names for the points -2, -3, -4, and -5.

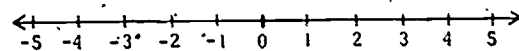
Explain that all the numbers to the left of zero are designated with a minus-sign and are called negative numbers. Ask how many negative numbers the children think there are, and whatever the number suggested, ask what the number to the left of it would be. Explain that there are just as many negative numbers as there are positive numbers (numbers to the right of zero). Do more examples with answers that are negative numbers:  $7 - 9$ ,  $7 - 20$ ,  $13 - 14$ ,  $21 - 30$ .

Have the children complete Worksheet 57.

Worksheet 57  
Unit 16

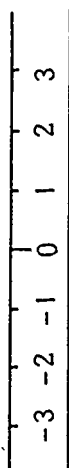
Name \_\_\_\_\_

Use the number line to help you find the answers.



$4 - 6 =$ <span style="border: 1px solid black; padding: 2px;">-2</span>	$1 - 6 =$ <span style="border: 1px solid black; padding: 2px;">-5</span>
$5 - 3 =$ <span style="border: 1px solid black; padding: 2px;">2</span>	$4 - 2 =$ <span style="border: 1px solid black; padding: 2px;">2</span>
$0 - 3 =$ <span style="border: 1px solid black; padding: 2px;">-3</span>	$1 - 0 =$ <span style="border: 1px solid black; padding: 2px;">1</span>
$2 - 5 =$ <span style="border: 1px solid black; padding: 2px;">-3</span>	$3 - 4 =$ <span style="border: 1px solid black; padding: 2px;">-1</span>
$4 - 7 =$ <span style="border: 1px solid black; padding: 2px;">-3</span>	$5 - 7 =$ <span style="border: 1px solid black; padding: 2px;">-2</span>

## Activity B



For this activity the children should have -15 to +25 number lines on their desks. Have them rotate the number line counter-clockwise from a horizontal position to a vertical position. Be sure that the positive numerals are above 0 and the negative numerals are below 0.

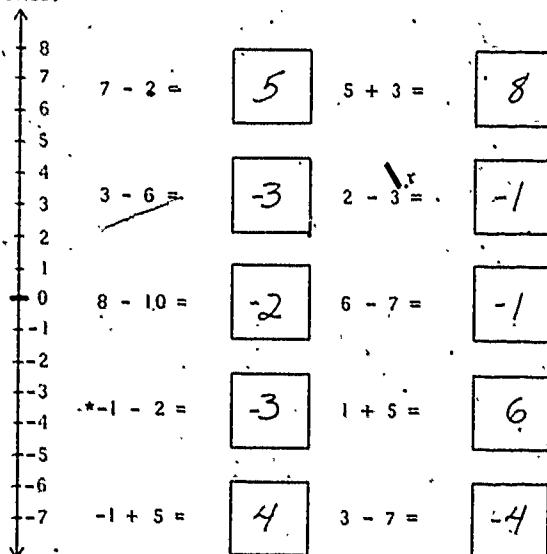
Explain to the children that they now have a number line like that on a thermometer. Very cold temperatures, such as those below zero, are represented by negative numbers. For example, 14 below 0 is written as -14. Give the children practice in adding and subtracting with this vertical number line. Point out that  $4 + 2$  means that they must start at 4 and move upward two units ( $4 + 2 = 6$ ) and  $4 - 6$  means they must start at 4 and move downward 6 units ( $4 - 6 = -2$ ). You may wish to use problems dealing with temperature, e.g.:  $17 - 10 = 7$ , etc.

Worksheet 58

Unit 16

Name \_\_\_\_\_

Use the vertical number line to help you fill in the boxes.



Have the children complete Worksheet 58.

### Activity C: Games

#### 1. "Take a Trip"

Draw a large number line on the floor. Mark off points about 12 inches apart and label them from  $-15$  to  $+15$ .

Divide the children into groups of five. One group will play at a time. Label several 3" x 5" cards with numerals from  $-1$  to  $+3$ . The cards should be shuffled. Each member of the group gets five moves. He draws five cards at random, starts at zero and steps along the number line as indicated by the numerals on his cards, in the order in which he drew them. The point where he is after the fifth move is recorded. You can decide beforehand as to what rule will determine the winner -- high score, low score, closest to zero, etc. Each time the children play, vary the rules for winning.

#### 2. "Captain, May I?"

Play this with a number line on the floor. Have the captain say the number of units and the direction of each child's steps. Give all children a turn at being captain.

## SECTION 5 TEACHING TOR TO USE OUR MONEY (Lesson 28)

This lesson introduces the relations among the different American coins. It reinforces the idea of grouping as the basis of place-value notation. Pennies, nickels, dimes, and a quarter are considered. You may wish to include work with half-dollars and dollars also, but this is optional.

When you use the parallel number lines to show relations among the coins, do not try to develop any understanding of the multiplicative relations involved. This is done in Unit 17, Introducing Multiplication and Division. Do not ask the children to make parallel number lines of their own.

Teaching Tor is a helpful device for going over material with which the children may already be familiar, so a segment of the Tor story is provided to introduce the lesson. Then, since this is the last lesson of the unit, another segment is provided at the end; it describes Tor's departure for his home.

### MATERIALS

- pennies, nickels, dimes, quarters (halves and dollars optional); or facsimiles from the coin sheet
- addition slide rules
- Worksheets 59, 60 and 61

### PROCEDURE

#### Activity A

#### TOR FINDS OUT ABOUT MONEY

Tommy reached into his pocket to pull out his handkerchief, and a penny fell to the floor. It made a tinkling sound, and Tor picked it up and looked at it curiously. "This seems to be a piece of copper," he said, "but it has a very interesting design. What is it?"

"It's money," Tommy said. "It's called a penny, Tor."

"What do you do with it? What is it used for?" Tor asked.

"It's for buying things."

"Buying?" Tor was puzzled. "What does 'buying' mean?"

"We go to the store and ask the man there for a piece of candy or a balloon or anything he has that we want. He gives us the candy or balloon, or whatever we want, and we give him the penny," Tommy said. "We just exchange the penny for what we want."

"If I gave the man in the store this penny, could I buy a big chair like the one you are sitting on?" Tor asked.

"Oh, no. That would cost dollars, not pennies."

"What are dollars?"

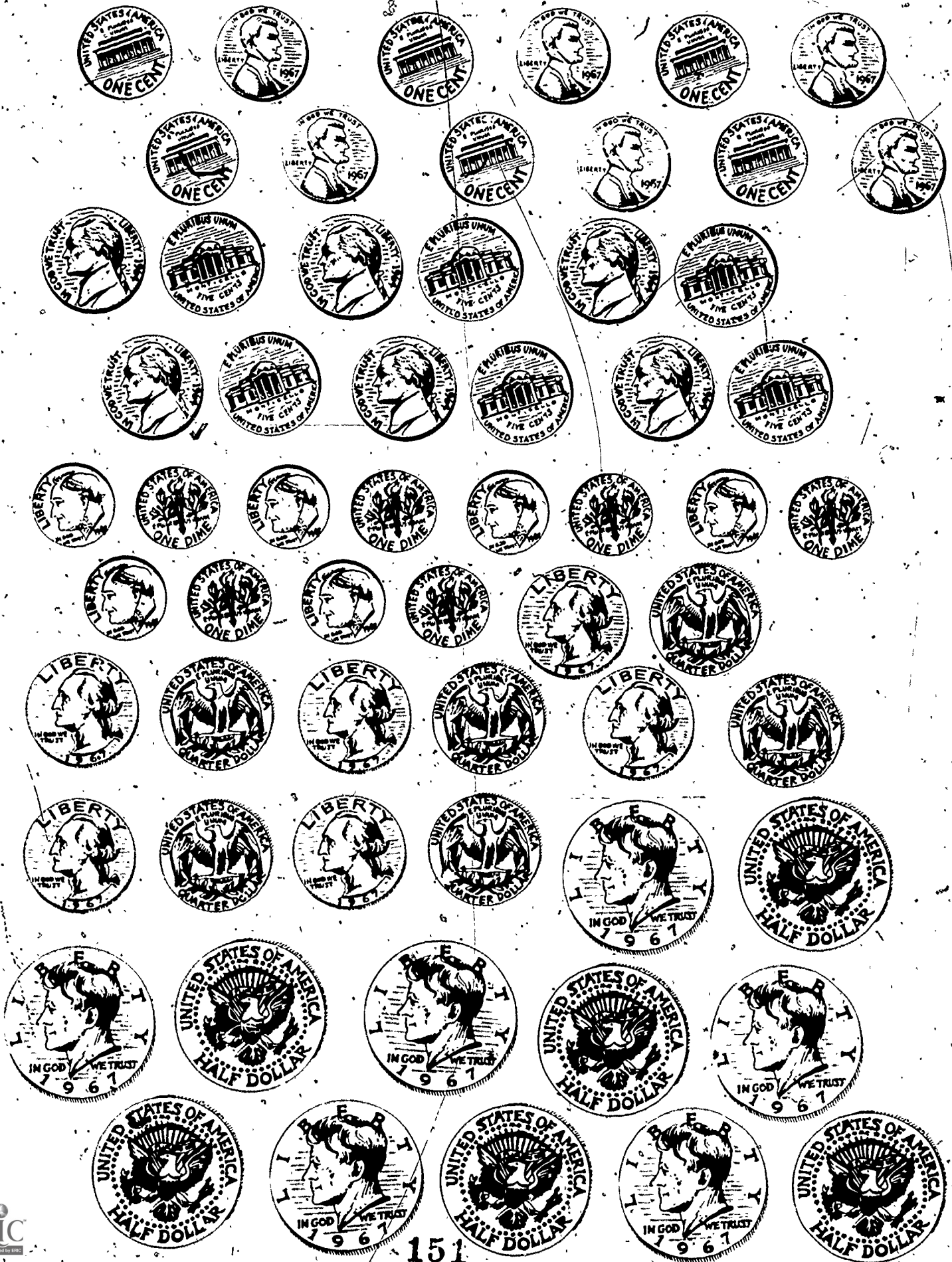
"Well, dollars are money too, but they buy more than pennies do. A dollar will buy as much as one hundred pennies could."

Miss Johnson spoke up. "Why don't we all help Tor find out how our money system works?"

So the children told Tor that people worked and were paid money, and that when they needed something, they could take the money to the store and exchange it for what they needed. The children explained that some things cost just a penny or a few pennies, but other things cost many dollars.

.....

Remind the children that Tor is always interested in how Earth People write things down. Write "1 penny" on the

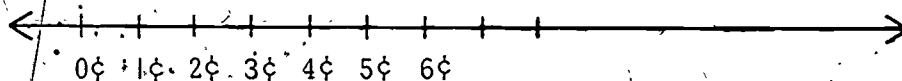




chalk board, and ask if there is another way this could be written. You should finally have on the chalk board the following:

$$1 \text{ penny} = 1 \text{ cent} = 1\text{¢}$$

With the children's help, develop a list on the chalk board with as many examples as are needed to establish this method of notation. Your list should include 5¢, 10¢, and 25¢.



IF WE WANT TO SOLVE A PROBLEM ABOUT MONEY,  
DO WE NEED THE CENTS SYMBOLS ON THE NUMBER  
LINE? (No.)

Erase the cents symbols you just wrote, and fill in more numerals to emphasize that this is just like any other number line, even though you are representing pennies?

#### Activity B

For this activity use real coins or the printed coins provided in this manual and in the Student Manuals.

TOR IS A VERY PRACTICAL FELLOW, AND HE WANTS TO  
KNOW IF EARTH PEOPLE GET TIRED CARRYING AROUND  
SO MANY PENNIES. WHAT SHALL WE TELL HIM?  
(We have other coins worth more than pennies.)

Have the children name all the denominations of money they can. Then show them a nickel and ask how many pennies could be exchanged for one nickel. On the chalk board write:

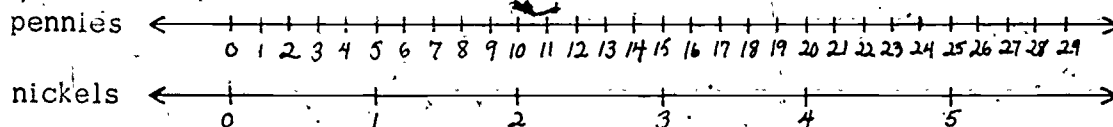
$$1 \text{ nickel} = 5 \text{ pennies} = 5\text{¢}$$



Take five pennies in one hand and five pennies in the other, and then put them together. Using repeated addition, develop the following table:

- 1 nickel = 5 pennies = 5¢
- 2 nickels = (5¢ + 5¢) = 10¢ = 10 pennies
- 3 nickels = (5¢ + 5¢ + 5¢) = 15¢ = 15 pennies
- 4 nickels = (5¢ + 5¢ + 5¢ + 5¢) = 20¢ = 20 pennies
- 5 nickels = (5¢ + 5¢ + 5¢ + 5¢ + 5¢) = 25¢ = 25 pennies

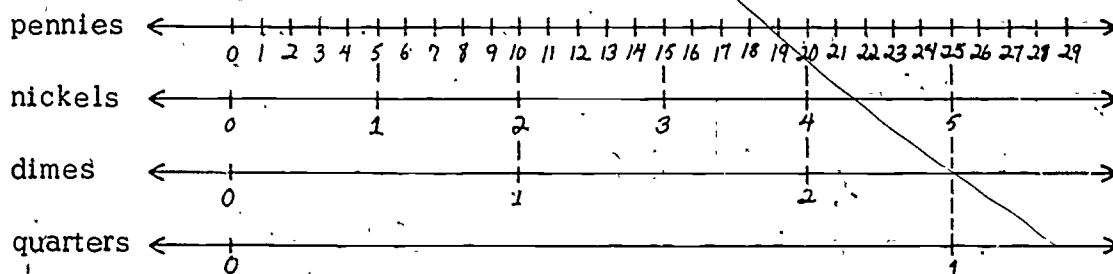
Near the top of the chalk board draw a number line labeled to 25. At the left, label the line "pennies." Below this line draw another line labeled "nickels." When you mark off the units on this line, be sure that your 0 is directly under the 0 on the pennies line and that your 1 is directly under the 5 on the first line, and so on.



Show the children a dime and ask how many pennies it is worth. Develop a table for dimes in the same way you did for nickels, continuing as long as you consider necessary.

Draw a number line below the first two, for dimes. Be sure your 1-point is directly below the 10-point on the penny line and the 2-point on the nickel line.

Show the children a quarter and ask how many pennies it is worth. Develop a table and then a number line as you did for the other coins. Your number lines should now look like this:



Worksheet 59  
Unit 16

Name \_\_\_\_\_

Use your addition slide rule or the number line to help you solve these problems.

$$17 \text{ pennies} + 9 \text{ pennies} = \underline{26} \text{ pennies} = \underline{26} \text{¢}$$

$$24 \text{ pennies} + 4 \text{ pennies} = \underline{28} \text{ pennies} = \underline{28} \text{¢}$$

$$63 \text{ pennies} + 8 \text{ pennies} = \underline{71} \text{ pennies} = \underline{71} \text{¢}$$

$$5\text{¢} + 13\text{¢} = \underline{18} \text{¢}$$

$$21\text{¢} + 9\text{¢} = \underline{30} \text{¢}$$

$$93\text{¢} + 4\text{¢} = \underline{97} \text{¢}$$

$$16\text{¢} + 4\text{¢} = \underline{20} \text{¢}$$

$$32\text{¢} + 10\text{¢} = \underline{42} \text{¢}$$

To illustrate the interrelations among the coins, draw dotted vertical lines as shown. Point to 5 on the first line and ask what coins could be used to represent the number of cents. (5 pennies or 1 nickel.) Repeat with 10, 15, etc. Continue until you think the children are ready for Worksheets 59, 60, and 61; and then have the worksheets completed. Do not have the children do the starred problems on Worksheets 60 and 61 unless you have presented the necessary class work.

After completing the worksheets, if the children wish to play with the printed coins in their Student Manuals, have them paste the sheet on a piece of light cardboard or construction paper and then cut out the coins.

Worksheet 60  
Unit 16

Name \_\_\_\_\_

$$10\text{¢} = \underline{1} \text{ dime} = \underline{2} \text{ nickels} = \underline{10} \text{ pennies}$$

$$20\text{¢} = \underline{2} \text{ dimes} = \underline{4} \text{ nickels} = \underline{20} \text{ pennies}$$

$$60\text{¢} = \underline{6} \text{ dimes} = \underline{12} \text{ nickels} = \underline{60} \text{ pennies}$$

$$* 50\text{¢} = \underline{1} \text{ half dollar} = \underline{2} \text{ quarters} = \underline{50} \text{ pennies}$$

$$* 100\text{¢} = \underline{2} \text{ half dollars} = \underline{4} \text{ quarters} = \underline{100} \text{ pennies}$$

$$15\text{¢} = \underline{3} \text{ nickels} = \underline{15} \text{ pennies}$$

$$35\text{¢} = \underline{7} \text{ nickels} = \underline{35} \text{ pennies}$$

$$45\text{¢} = \underline{9} \text{ nickels} = \underline{45} \text{ pennies}$$

Worksheet 61  
Unit 16

Name \_\_\_\_\_

$$1 \text{ dime} + 1 \text{ quarter} = \underline{10} \text{¢} + \underline{25} \text{¢} = \underline{35} \text{¢}$$

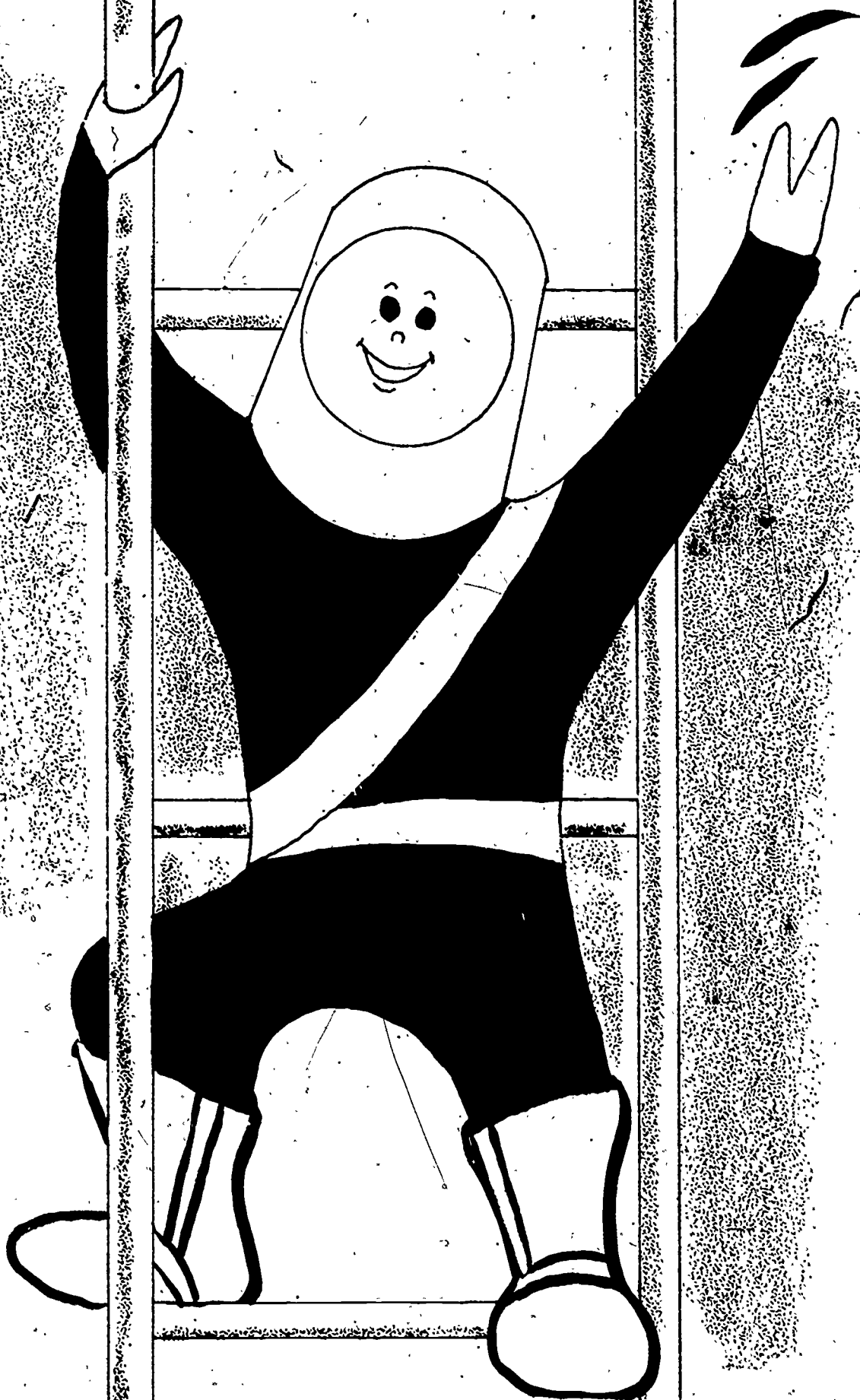
$$2 \text{ nickels} + 2 \text{ dimes} = \underline{10} \text{¢} + \underline{20} \text{¢} = \underline{30} \text{¢}$$

$$4 \text{ pennies} + 1 \text{ quarter} = \underline{4} \text{¢} + \underline{25} \text{¢} = \underline{29} \text{¢}$$

$$3 \text{ pennies} + 6 \text{ pennies} = \underline{3} \text{¢} + \underline{6} \text{¢} = \underline{9} \text{¢}$$

$$* 1 \text{ half dollar} + 1 \text{ dime} = \underline{50} \text{¢} + \underline{10} \text{¢} = \underline{60} \text{¢}$$

$$* 2 \text{ quarters} + 3 \text{ nickels} = \underline{50} \text{¢} + \underline{15} \text{¢} = \underline{65} \text{¢}$$



## TOR SAYS GOODBYE

At the end of the lesson about coins, Miss Johnson noticed that Tor wanted to say something. He was waving his hand around in the air for her attention.

"What is it, Tor?" she asked. "Do you want the children to explain more about our money?"

"No," Tor said, "What I want to say is that I have to go back to Titan now -- at least for a while -- and I have to say goodbye. Thank you all for teaching me so much."

Tommy thought he saw tears in Tor's eyes. "Don't go, Tor," he said, "we can still teach you about many, many more things."

Then the children all gathered around Tor, begging him not to leave. Sally said, "We like you so much, Tor!" and Joe said, "It's so much fun learning with you," and John said, "Why do you have to go, anyway?"

Tor said, "I have to go now, while I still have enough food and other supplies left for my journey. I have never learned to eat your food, you know, even though I have tried."

"Tor's right," Mary said, "he can't even eat cookies. They don't agree with him at all."

The children groaned. It seemed very sad that their friend could not eat cookies.

Miss Johnson said, "We want to thank you, Tor, for coming to visit us. It has been a great pleasure to have you here. We appreciate it so much that we certainly will not try to make you stay when we understand why you have to go home."

Tommy said, "Tor, how soon do you have to leave?"

"It will take me a little while to check my space ship," Tor said, "but then I must take off."

"Well, check your ship and then come back -- at least for a minute," Tommy said.

While Tor was checking his ship, inside and out, the children and Miss Johnson were feverishly planning how they should say goodbye to their little friend.

"I'm going to draw a picture for him," Sally said, and she got busy with her crayons.

"I'm going to give Tor my shiny penny," Tommy said.

"Let's put whatever we can give Tor on the display table," Joe said, "and let him take his pick."

"And when he leaves we will sing 'Happy Journey' to him," Eddie said.

"I never heard of that song," Jack said. "How does it go?"

"It's just like 'Happy Birthday,' only you sing, 'Happy Journey, dear Tor' instead."

"Good idea!" Jack said, and the other children agreed.

When Tor returned, the children were all waiting for him. Tommy showed Tor the presents on the display table. "You can have whatever you want to take back to Titan with you," Tommy explained. "Some of the children drew pictures for you; others gave small things they thought your friends on Titan might like to see. We know you don't have space for anything very large."

Tor's eyes shone as he looked at the gifts. "Pictures," he exclaimed happily, "and pencils! Paper clips! Crayons! Chalk! A shiny penny! Oh, what a wonderful selection! What fun I will have showing these things back home! My friends on Titan have never seen anything like them!"

"You will need this box to put your presents in," Sally said, as she gave him an empty carton.

"My, you think of everything!" Tor said. Then he began to pack the box. The children clustered around, watching. Tor admired each picture before he placed it in the bottom of the box. Next, he carefully placed all the small gifts on top of the pictures. Then, much to everyone's surprise, Tor put in an addition slide rule, an abacus, and even a beam balance that just happened to be left on the display table.

Tor said, "Thank you" again and again and the children said, "Hurry back, Tor!" "Don't forget us, Tor!" and "Say hello to your friends on Titan!"

When everything was packed, Tor picked up the box and went to the window. The children followed, singing the Happy Journey song. They watched and sang as Tor sprayed himself out to his space ship. They waited at the window while he put the box inside the ship. Then he came out once more, and stood on the ladder to wave goodbye.

Everyone waved back, and shouted "Goodbye" and "Good luck." And everyone watched as the little space ship took off. They watched and watched until the ship was only a little speck in the sky. Finally, when they could not even see the speck anymore, Mary said, "Miss Johnson, did you expect Tor to take the

abacus, the slide rule and the beam balance?"

"No, they just happened to be left there on the table." Miss Johnson blushed. The children smiled, enjoying the joke.

Then Sammy said, "I'm glad you left them there. I think it was lucky, because anyone could see that Tor really wanted them."

"I'm glad, too," Miss Johnson said. "I think Tor will make good use of them."

Tommy said, "Miss Johnson, do you think Tor will come back back to visit us again?"

"I don't know, Tommy. He seems to have a very fast and efficient space ship, but he never could explain how it was made or how long it took him to get here."

"I hope he does come back sometime," Sally said. "It was fun explaining things to him."

Tommy said, "Well, who knows? Maybe some day we will have such a good space ship that we can go and visit him."

Some of the children laughed, but Sammy said, "That just might be possible, you know." His remark seemed to brighten every face in the room.

